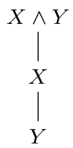
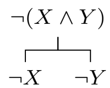
SL Rules

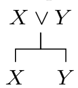
$\wedge$ Decomposition



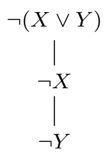
$\neg\wedge$ Decomposition



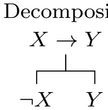
$\vee$ Decomposition



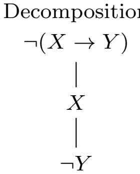
$\neg\vee$ Decomposition



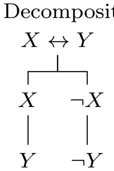
$\rightarrow$ Decomposition



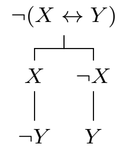
$\neg\rightarrow$ Decomposition



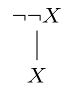
$\leftrightarrow$ Decomposition



$\neg\leftrightarrow$ Decomposition



$\neg\neg$ Decomposition



PL Rules

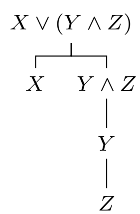
Notations

Substitution Instance

Robotnese: for any expression Φ, a substitution instance $\Phi^{\beta}/_{\alpha}$ refers to the formula obtained by replacing all occurrence of α in Φ by some β not already in Φ.
Human-speak: Consider $\forall x Px$. Let $\Phi = Px$. A substitution instance of Px could be Pc, where we $\alpha = x$ and $\beta = c$ - All occurrences of x are replaced with c.

Root

The lowest node. For instance, $X \vee (Y \wedge Z)$ is the root for the tree below:



Path

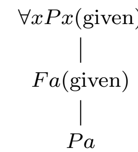
A (directed) path is a non-branching series of connected and nodes. Take the tree above, there two directed paths in this tree. One from the root to X, and the other one goes from the root to Z. In our context, we only talk about directed path.

PL Rules

$\forall$ Decomposition If a non-decomposed (not marked with an X) PL sentence $\forall\alpha\Phi$ occurs on an open path(i.e., branch), you may:

1. If some constant β_0 is already present on that path, add a node on that path with the PL sentence with the substitution instance $\Phi^{\beta}/_{\alpha}$.
2. For all constants $\beta_1, ..\beta_n$ on that path, add a new substitution instance as you did for step 1.
3. If no constant exists on that path, pick some β and add a node on that path with the substitution instance $\Phi^{\beta}/_{\alpha}$.
4. In either case, *don't* check the $\forall\alpha\Phi$.
5. If a new constant β_{n+1} is later introduced, instantiate $\forall\alpha\Phi$ by writing $\Phi^{\beta_{n+1}}/_{\alpha}$. This might produce infinite trees.

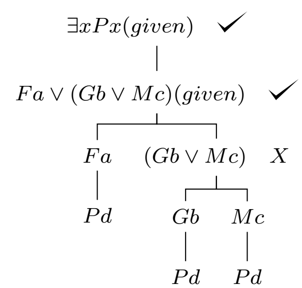
Example:



$\exists$ Decomposition If a non-decomposed (not marked with an ✓) PL sentence $\exists\alpha\Phi$ occurs on an open path(i.e., branch), you may:

1. You must pick some β constant that has not occurred *at all* and add a node on that path with the substitution instance $\Phi^{\beta}/_{\alpha}$.
2. Add $\Phi^{\beta}/_{\alpha}$ too all open paths.
3. *Do* check the $\exists\alpha\Phi$.

Example:



$\neg\forall$ Decomposition If a non-decomposed (not marked with an ✓) PL sentence $\neg\forall\alpha\Phi$ occurs on an open path(i.e., branch), you may:

1. Add $\exists\neg\alpha\Phi$ too all paths that contain $\neg\forall\alpha\Phi$
2. Check the original node

$\neg\exists$ Decomposition If a non-decomposed (not marked with an X) PL sentence $\neg\exists\alpha\Phi$ occurs on an open path(i.e., branch), you may:

1. Add $\forall\neg\alpha\Phi$ too all paths that contain $\neg\exists\alpha\Phi$
2. Check the original node