Problem-Set 1.3: Substitution of Logical Equivalents

Background The logical laws are extremely useful, because logical equivalents are *mutually substitutable*, as summarized by the following principle:

The Law of Substitution of Logical Equivalents (SLE): For any two logically equivalent sentences X and Y, if X occurs as a proper substring of some longer sentence Z. A new sentence Z' that is logically equivalent to Z can be constructed by substituting Y for X in Z.

Example SLE allows us to carry out quasi-algebraic operations on SL sentences. Here's an example. Suppose we want to prove that $\neg[(\neg A \vee \neg B) \wedge (\neg A \vee B) \equiv A \wedge (B \vee \neg B)]$. We do it line by line as follows:

$$\neg[(\neg A \vee \neg B) \wedge (\neg A \vee B)] \tag{1}$$
$$\neg(\neg A \vee \neg B) \vee \neg(\neg A \vee B) \quad \text{DeMorgan's Law} \tag{2}$$
$$(\neg\neg A \wedge \neg\neg B) \vee (\neg\neg A \wedge \vee B) \quad \text{DeMorgan's Law} \tag{3}$$
$$(A \wedge B) \vee (A \wedge \vee B) \quad \text{Double Negation} \tag{4}$$
$$A \wedge (B \vee \neg B) \quad \text{Distribution} \tag{5}$$

Problems Prove the following equivalence relations using SLE.

1. $B \vee \neg A \equiv \neg(A \wedge \neg B)$
2. $(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$
3. $A \wedge (\neg\neg C \vee B) \equiv (A \wedge C) \vee (A \wedge B)$
4. $\neg[(A \wedge \neg B) \vee (C \wedge \neg B)] \equiv (\neg A \wedge \neg C) \vee B$
5. Create your own logical equivalents and prove them. It must have at least 5 lines and use at least two different laws.