

SL Rules

Modus Ponens ($\rightarrow E$)

$$\begin{array}{|l} P \rightarrow Q \\ P \\ \hline Q \end{array}$$

Conjunction Introduction ($\wedge I$)

$$\begin{array}{|l} P \\ Q \\ \hline (P \wedge Q) \end{array}$$

Conjunction Elimination ($\wedge E$)

$$\begin{array}{|l} (P \wedge Q) \\ \hline P \\ Q \end{array}$$

Disjunction Introduction ($\vee I$)

$$\begin{array}{|l} (P) \\ \hline P \vee Q \end{array}$$

Disjunction Elimination ($\vee E$)

$$\begin{array}{|l} P \vee Q \\ \neg P \\ \hline Q \end{array}$$

Biconditional Elimination ($\leftrightarrow E$)

$$\begin{array}{|l} (P \leftrightarrow Q) \\ \hline P \rightarrow Q \\ Q \rightarrow P \end{array}$$

Negation Elimination ($\neg E$)

$$\begin{array}{|l} \neg\neg P \\ P \\ \hline \end{array}$$

Hypothetical Rules

Reiteration (R)

*1

$$\begin{array}{|l} P \\ \hline P \end{array}$$

Conditional Introduction ($\rightarrow I$)

$$\begin{array}{|l} | n. P \\ | \vdots \\ | m. Q \\ \hline P \rightarrow Q \end{array}$$

Reductio ad Absurdum (RAA)

$$\begin{array}{|l} | n. P \\ | \vdots \\ | m. Q \\ | m' \neg Q \\ \hline \neg P \end{array}$$

Quantifier Rules

Universal Introduction ($\forall I$)

$$\begin{array}{|l} \Phi\alpha \\ \triangleright \forall\chi\Phi\chi \end{array}$$

provided that

(1) α does not occur in an open assumption

(2) α does not occur in $\forall\chi\Phi\chi$

Universal Elimination ($\forall E$)

$$\begin{array}{|l} \forall\chi\Phi\chi \\ \triangleright \Phi\alpha \end{array}$$

Existential Introduction ($\exists I$)

$$\begin{array}{|l} \Phi\alpha \\ \triangleright \exists\chi\Phi\chi \end{array}$$

Existential Elimination ($\exists E$)

$$\begin{array}{|l} \exists\chi\Phi\chi \\ | \Phi\alpha \\ | \Psi \\ \hline \triangleright \Psi \end{array}$$

provided that

(1) α does not occur in an open assumption

(2) α does not occur in $\exists\chi\Phi\chi$

(3) α does not occur in Ψ

Quantifier Negation (QN):

$$\neg\forall\chi\Phi\chi \equiv \exists\chi\neg\Phi\chi$$

$$\neg\exists\chi\Phi\chi \equiv \forall\chi\neg\Phi\chi$$

Identity Rules

Identity Introduction ($=I$)

$$\begin{array}{|l} \hat{\alpha} = \hat{\alpha} \end{array}$$

Identity Elimination ($=e$)

$$\begin{array}{|l} \alpha = \beta \\ \Phi\alpha \\ \hline \triangleright \Phi\beta \end{array}$$

¹Not a hypothetical rule but often used together.