Last problem set. Try to contain your sadness.

1 Consider this partial model:

- UD: the set of all Martians - you can interpret this set in anyway you like, i.e., you can stipulate how many Martians there are, but it cannot be {}.
- Lxy: x loves y.
- Pxy x is a parent of y.

Note: You cannot introduce more predicates. Use only L, P, and identity.

1.1 Translate the argument in such a way that would render it VALID, and then prove its validity using a tree.

Every Martian loves some Martian, self excluded. All Martians love their children. Therefore, no parent of a Martian is that same Martian.

(Hint: the first premise is ambiguous.)

1.2 The following argument is INVALID. Translate it and then provide a model to prove its invalidity. Use a tree if you like, but it's optional if you can figure a model without it.

Every Martian has exactly two parents. Therefore every Martian has exactly four grandparents.

Note:

- If you need more variables, keep in mind that you can have an infinite number of variables by introducing indices, i.e., $x_1, x_2, ...$
- You MUST not introduce a new predicate for the grandparent relation.
- If the translation is so long you feel like pulling your hair out, you are on the right track.

Deduction Prove the following:

1.1 $\vdash \neg a = b \leftrightarrow \neg b = a$

1.2 $\vdash \forall x \forall y \forall z((x = y \wedge y = z) \rightarrow x = z)$

Practice for Test 4
Translation Practice

1. Everyone knows oneself and no one else.
2. There are exactly two persons who knows Bob.
3. Bob is the loneliest American
4. The son of Bob is happy.
5. The smartest student is the best student.
6. The early bird gets the worm.

Translate and evaluate the following arguments.

1. Some Martians are parents. Some Martians are children. Therefore, there are at least two Martians.
2. There is exactly one Martian parent. There is exactly one Martian lover. Nothing is both a lover and a parent. Therefore, more than two Martians are either a parent or a lover.
3. No Martian has exactly three parents. All Martians have more than one parent. Therefore, all Martians have two parents.

Prove the following deductions:

1. $\forall z(Gz \rightarrow \forall y(Ky \rightarrow Hzy)), ((Ki \wedge Gj) \wedge i = j) \vdash Hii$
2. $\forall x(x = a \rightarrow Fx) \vdash Fa$
3. $a = b \vdash \forall x(a = x \leftrightarrow b = x)$
4. $\exists x \forall y(Py \leftrightarrow y = x)$
5. $\vdash \forall x(Px \leftrightarrow \exists y(x = y \wedge Py))$
6. $\vdash \exists x \exists y(Fx \wedge \neg Fy) \rightarrow \exists x \exists y(x \neq y)$
7. $\vdash \forall x \forall y(x = y \rightarrow y = x)$
8. $\vdash \forall x \forall y((Fx \wedge \neg Fy) \rightarrow x \neq y)$

Model-Theoretic Semantics Determine the truth values of the following sentences on the model below. (It may help to think first about what they mean in English, or in your favorite natural language.)

1. $U = \{1, 2, 3, 4, ...\}$ (i.e., the set of all positive integers)
2. $G = \{(x, y)|x > y\}$
3. $P = \{(x, y, z)|x + y = z\}$

$\forall x \forall y \forall z((Gxy \wedge Gyz) \rightarrow x \neq z)$

$\forall x \forall y \exists z((Pxyz \wedge x \neq z) \wedge y \neq z)$